

Pozo cuadrado

CT I H_I

● STATIONARY STATES OF A PARTICLE IN ONE-DIMENSIONAL SQUARE POTENTIALS

2-c. Bound states: square well potential

α. Well of finite depth

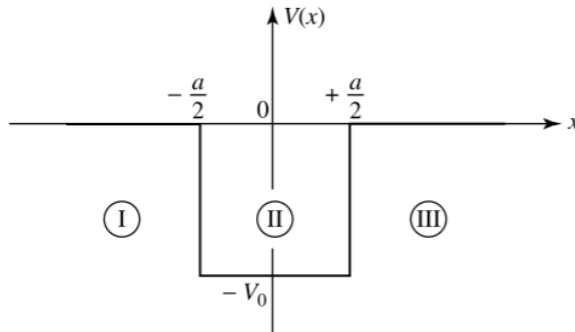


Figure 4: Square well potential.

We shall limit ourselves to studying the case $-V_0 < E < 0$ (the case $E > 0$ was included in the calculations of the preceding section 2-b-α).

In regions I ($x < -\frac{a}{2}$), II ($-\frac{a}{2} \leq x \leq \frac{a}{2}$), and III ($x > \frac{a}{2}$) shown in Fig. 4, we have respectively:

$$\varphi_{\text{I}}(x) = B_1 e^{\rho x} + B'_1 e^{-\rho x} \quad (36\text{-a})$$

$$\varphi_{\text{II}}(x) = A_2 e^{ikx} + A'_2 e^{-ikx} \quad (36\text{-b})$$

$$\varphi_{\text{III}}(x) = B_3 e^{\rho x} + B'_3 e^{-\rho x} \quad (36\text{-c})$$

with

$$\rho = \sqrt{-\frac{2mE}{\hbar^2}} \quad (37)$$

Queda encontrar la relación entre g, k y las otras constantes para satisfacer las condiciones de frontera.

$$k = \sqrt{\frac{2m(E + V_0)}{\hbar^2}} \quad (38)$$

Since $\varphi(x)$ must be bounded in region I, we must have:

$$B'_1 = 0 \quad (39)$$

The matching conditions at $x = -\frac{a}{2}$ then give:

$$\begin{aligned} A_2 &= e^{(-\rho+ik)a/2} \frac{\rho+ik}{2ik} B_1 \\ A'_2 &= -e^{-(\rho+ik)a/2} \frac{\rho-ik}{2ik} B_1 \end{aligned} \quad (40)$$

and those at $x = a/2$:

$$\frac{B_3}{B_1} = \frac{e^{-\rho a}}{4ik\rho} [(\rho + ik)^2 e^{ika} - (\rho - ik)^2 e^{-ika}]$$

$$\frac{B_3'}{B_1} = \frac{\rho^2 + k^2}{2k\rho} \sin ka \quad (41)$$

But $\varphi(x)$ must also be bounded in region III. Therefore, it is necessary that $B_3 = 0$, that is:

$$\left(\frac{\rho - ik}{\rho + ik}\right)^2 = e^{2ika} \quad (42)$$

Since ρ and k depend on E , equation (42) can only be satisfied for certain values of E . Imposing a bound on $\varphi(x)$ in all regions of space thus entails the quantization of energy. More precisely, two cases are possible:

(i) if:

$$\frac{\rho - ik}{\rho + ik} = -e^{ika} \quad (43)$$

we have:

$$\frac{\rho}{k} = \tan\left(\frac{ka}{2}\right) \quad (44)$$

$$\frac{\rho - ik}{\rho + ik} = -e^{ika} = -\cos ka - i \sin ka$$

$$\frac{\rho - ik}{\rho + ik} \cdot \frac{\rho - ik}{\rho - ik} = \frac{\rho^2 - 2ik\rho - k^2}{\rho^2 + k^2} \Rightarrow +\cos ka = \frac{\rho^2 - k^2}{\rho^2 + k^2}$$

$$+ \sin ka = \frac{2k\rho}{\rho^2 + k^2}$$

$$\tan(2\theta) = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$\tan ka = \frac{2k\rho}{k^2 - \rho^2} = \frac{2 \rho/k}{1 - \rho^2/k^2} \Rightarrow \tan\left(\frac{ka}{2}\right) = \rho/k$$

Análogo $\frac{\rho - ik}{\rho + ik} = e^{ika} \Rightarrow -\cot\left(\frac{ka}{2}\right) = \rho/k$

